3-D Fractal Characterization of Tumors from a Computer Tomography Scan

Ernesto Cortés Pérez [1], Tomás Morales Acoltzi [2], Francisco Viveros Jiménez [3]

[1] Departamento de Ingeniería en Computación, Universidad Del ISTMO, UNISTMO, Campus Tehuantepec, Santo Domingo Tehuantepec, Oaxaca, Mexico
neto_144@bianni.unistmo.edu.mx

[2] Centro de Ciencias de la Atmósfera, UNAM, Circuito Exterior, CU, México DF CP 04510.
acoltzi@atmosfera.unam.mx

[3] Departamento de Licenciatura en Informática, Universidad Del ISTMO, UNISTMO, Campus Ixtepec, Avenida Universidad S/N, Ciudad Ixtepec Oaxaca, CP 70110, Mexico
fviveros@bianni.unistmo.edu.mx

Abstract. The use of the Box-Counting method (BC) to calculate the fractal dimension of invasive pathologies in the human body is proposed in this paper. BC was applied to calculate the fractal dimension of a tumor from medical images of human brains with cancer. The BC test required additional image processing algorithms and 3-D reconstruction software for the processing of a sample area. The BC results were used to determine the size and volume of a tumor; this allows an oncologist to perform a more informed diagnostic.

Keywords: Fractal dimension, Box-Counting, cancer, medicine, tumor size, and pathology size.

1 Introduction

Saving human lives is the primary goal of medicine. Cancer is an invasive disease that is responsible for many deaths annually worldwide. For defeating cancer, an oncologist needs to know the following tumor characteristics:

- Specific type.
- Dimensions.
- Location.
- Internal organs affected.
- Cancer stage.
- Appearance.
- Growth rate.

© G. Sidorov (Ed.)
Advances in Artificial Intelligence: Algorithms and Applications
Research in Computing Science 40, 2008, pp. 95-106

Knowledge of these features allows an oncologist to estimate the operability of the tumor and prognosis of the patient.

Medical technology has improved during the last century. Technology like X-rays, ultrasound, magnetic resonance imaging (MRI) and computer tomography (CT) allows a physician to obtain an internal image of the body. This technology can even show brain functioning and structure. However, it is still difficult for a doctor to obtain an accurate measurement of an invasive pathology without an invasive surgical procedure. For this reason, many investigators have been using computer programs to help doctors to measure and identify more accurately an invasive pathology [1] [2]. This paper shows how Box-Counting method, which is widely used for calculation of fractal dimension, can be used to measure the size of a tumor in a human brain.

Box-Counting needs a digital filtered image for its performance. The process for obtaining this input is explained briefly later in this paper. Figure 1 shows an example of a CT scan of a patient with a brain tumor.

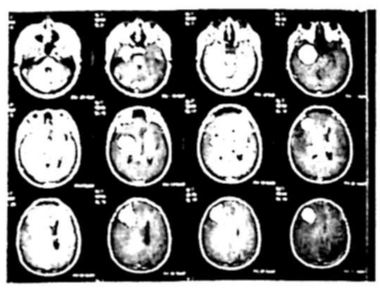

Fig. 1. CT scan of a brain with tumor.

2 Box–Counting Method

This section explains the process for calculating the dimension of an object with Box-Counting. First, the basic geometrical concepts are explained. Next, Box-Counting is explained. Finally, the measurement of teh size of an object with box-counting is shown by using exponential laws.

2.1 Traditional and Fractal Geometry Concepts

Traditional geometry is the sub-discipline of mathematics that studies the features and measurement of elements like points, lines, curves, planes, figures and volumes. However, traditional geometry can not represent the shapes found in nature like mountains, animals, clouds, leaves, trees, etc. Fractal geometry provides a mathematical model for these complicated natural forms (also called abstract forms).

In geometry, the dimension of a space is defined as the minimum number of coordinates needed to define every point within it. All forms have dimensions: points have none, lines have one dimension, surfaces have two dimensions and volumes have three dimensions. For example: two dimensions are required to represent a rectangle, a cube requires three dimensions, etc. This concept of dimension is called topological dimension.

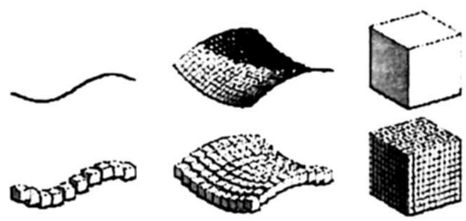

Fig. 2 Geometric shapes: line, surface and cube.

Surfaces like a lung, a brain or a tumor have three dimensions therefore the unevenness of an object can be considered as increment in a specific dimension. Rough curves have between one and two dimensions. Rough surfaces have between two and three dimensions. This concept is called fractal dimension. Fractal dimension is a precise parameter with which we measure the conceptual and visual complexity of an object. Using fractal dimension, the volume of an irregular object can be calculated.

2.2 Box-Counting Insight Explanation

Box-Counting is a method used to obtain the fractal dimension of an object. The Box-Counting steps are:

1. Draw a rectangular grid over the image that contains the object to identify. Each box will have a width and height of r. The grid will have N boxes.
2. Count the boxes that contain the object.

The relation between N and r is given in eq. 1.

$$N(1) = 1$$

$$N\left(\tfrac{1}{2}\right) = 4 = \frac{1}{\left(\frac{1}{4}\right)} = \left(\frac{1}{\left(\frac{1}{2}\right)}\right)^2$$

$$N\left(\tfrac{1}{4}\right) = 16 = \frac{1}{\left(\frac{1}{16}\right)} = \left(\frac{1}{\left(\frac{1}{4}\right)}\right)^2$$

$$N\left(\tfrac{1}{8}\right)=64=\frac{1}{\left(\frac{1}{64}\right)}=\left(\frac{1}{\left(\frac{1}{8}\right)}\right)^2$$

$$N(r)=\left(\frac{1}{r}\right)^2 \tag{1}$$

The precision of the technique depends directly on the size and number of boxes. A large number of smaller boxes will give a more accurate result. However, more boxes imply more image processing.

2.3 Exponential Laws

Equation 2 represents the relation between N and r for a d dimensional figure.

$$N(r)=\left(\frac{1}{r}\right)^d \tag{2}$$

Equation 3 is obtained by using exponential laws in eq. 2.

$$N(r)=k\left(\frac{1}{r}\right)^d \tag{3}$$

Equation 4 is obtained by applying logarithm laws to eq. 3.

$$\log(N(r))=\log(k)+\log\left(\left(\tfrac{1}{r}\right)^d\right)=d\log\left(\tfrac{1}{r}\right)+\log(k) \tag{4}$$

Equation 5 is obtained applying the limit of r when $\lim_{r\to 0}$. This deduction is made with an expectation of a better estimation.

$$d=\lim_{r\to 0}\frac{\log(N(r))}{\log\left(\frac{1}{r}\right)} \tag{5}$$

If the limit exists in eq. 5, then d can be calculated. However, the limit operation is complex. Equation 6 shows a valid approximation.

$$\log(N(r))=d\log\left(\frac{1}{r}\right)+\log(k) \tag{6}$$

Equation 7 is obtained by replacing the symbols of the linear equation in slope intersect form with eq. 6.

$$y=mx+b \tag{7}$$

where m represents the dimension.

In equation 7, b is the point, where the line intersects with y. A plot of $\log(N(r))$ against $\log(\frac{1}{r})$ must be approximated to the line with slope m. This approximation is called log-log approximation and is commonly used to find the dimension by Box-Counting method.

3 3-D Digital Reconstruction of a Brain Tumor from CT Scans

3-D digital reconstruction of a patient's head and a brain tumor is required before using the Box-Counting Method. This reconstruction gives a good visual representation of the size and position of the pathology.

Three processes are necessary to perform a 3-D digital reconstruction:

1. Segmentation [3] of the desired object from CT scans with Sobel operator [12].
2. Reconstruction of the 3-D model with Delaunay's triangulation [9].
3. Rendering of the resulting model.

3.1 Reconstruction of the Patient's Head

The first step for reconstructing the head is identification of the head's border in the sample images. 20 images form the sample in this research. Figure 3 shows an example of segmentation of a sample after using binary filtering and border detection with Sobel operator [12].

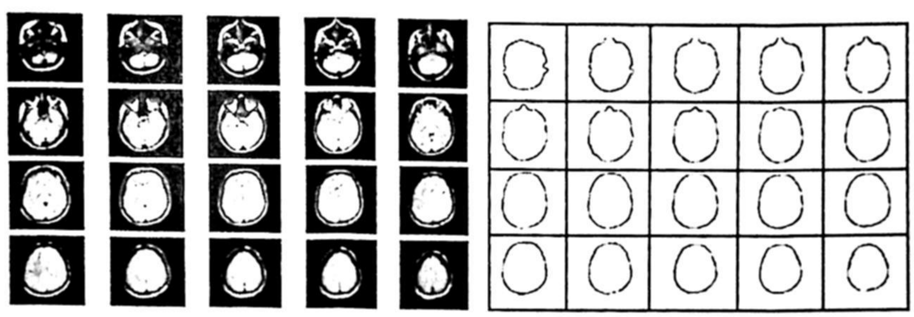

Fig. 3 Segmentation of the sample with Sobel operator.

Once the border is identified in the sample, the images will be superimposed in a 3-D space, as shown in Fig. 4.

Fig. 4 Borders from sample are superimposed in a 3-D space.

Next, Delaunay triangulation is used to create a non-structured grid. This process maximizes the interior angles with great precision (minimal rounding errors). Figure 5 shows the result of Delaunay triangulation in the borders obtained from the sample.

Fig. 5 Results from Delaunay triangulation process.

A rendering process [2] is executed over the grid obtained with Delaunay triangulation. Rendering is a process that creates an image taking environmental effects into account. Figure 6 shows the results of the rendering process.

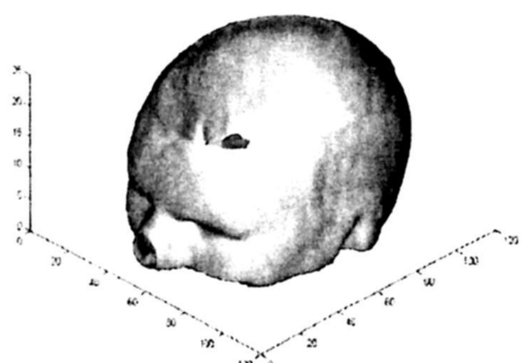

Fig. 6 Results of the rendering process of the patient's head.

3.2. Reconstruction of the Brain Tumor

The process of reconstruction of the brain tumor is exactly the same as the one used on the reconstruction of the patient's head. The first step is applying the segmentation process [3] to the sample. Figures 7, 8 and 9 show the results of the segmentation process.

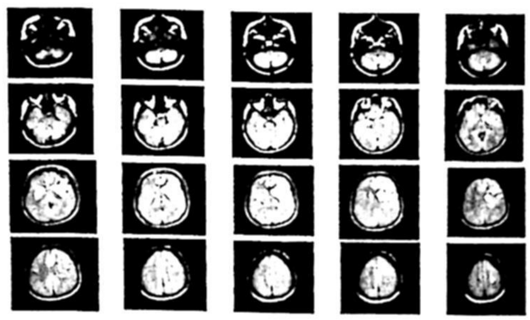

Fig. 7 Segmentation process.

Fig. 8 Segmentation of the pathology under examination

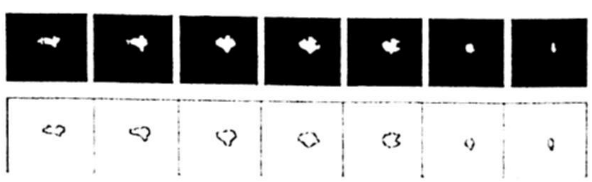

Fig. 9 Tumor borders obtained with Sobel operator.

Finally, the reconstruction of the tumor is applied. The rendering process must be executed taking the patient's head into consideration. In this way, useful information like location, size and internal organs affected is given to the specialist. Figure 10 shows an example. The rendered image can be used by specialists to decide if an operation is viable.

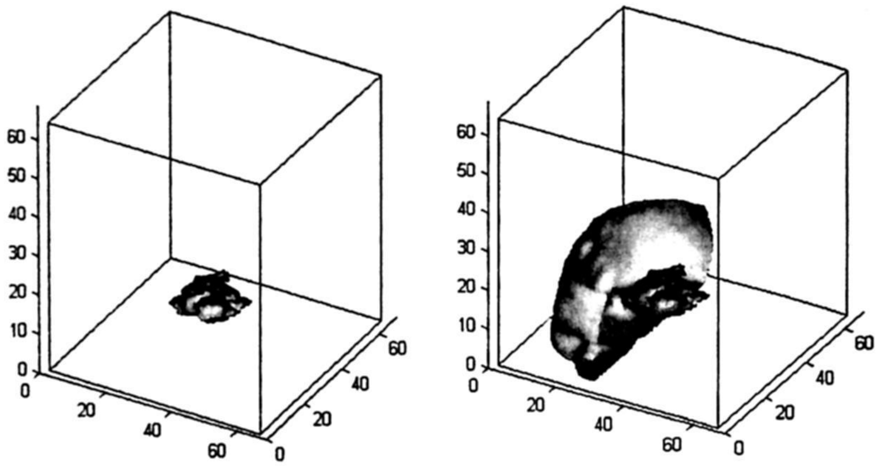

Fig. 10 3-D representation of the tumor.

4 Calculus of the Dimension of the Pathology in a 3-D Space

Measuring the brain tumor with Box-Counting becomes possible after the 3-D reconstruction. Because the object is 3 dimensional, the boxes are going to be cubes and $d = 3$, then $N(r)=\left(\frac{1}{r}\right)^3$.

Box-Counting is an iterative process. In each iteration, the method counts the boxes that contain the pathology. The first iteration has an r value of 1, because $\lim_{r \to 0}$, r is decremented in each cycle. Table 1 show the values obtained in 4 sample iterations of the Box-Counting method. In this table, n is the real depth, width and height of the boxes. Table 1 values consider a 3-D space of *64 x 64 x 64*.

Table 1. Sample iterations of the Box-Counting method

Iteration	n	r	N(r)
1	64	1	1/(1/1) ^ 3 = 1
2	32	1/ 2 = 0.5	1/(1/2) ^ 3 = 8
3	16	1/4 = 0.25	1/(1/4) ^ 3 = 64
4	8	1/8=0.125	1/(1/8) ^ 3 = 512

Figure 11 shows a 3-D image of the boxes proposed in table 1. Each iteration has more boxes and therefore more precision.

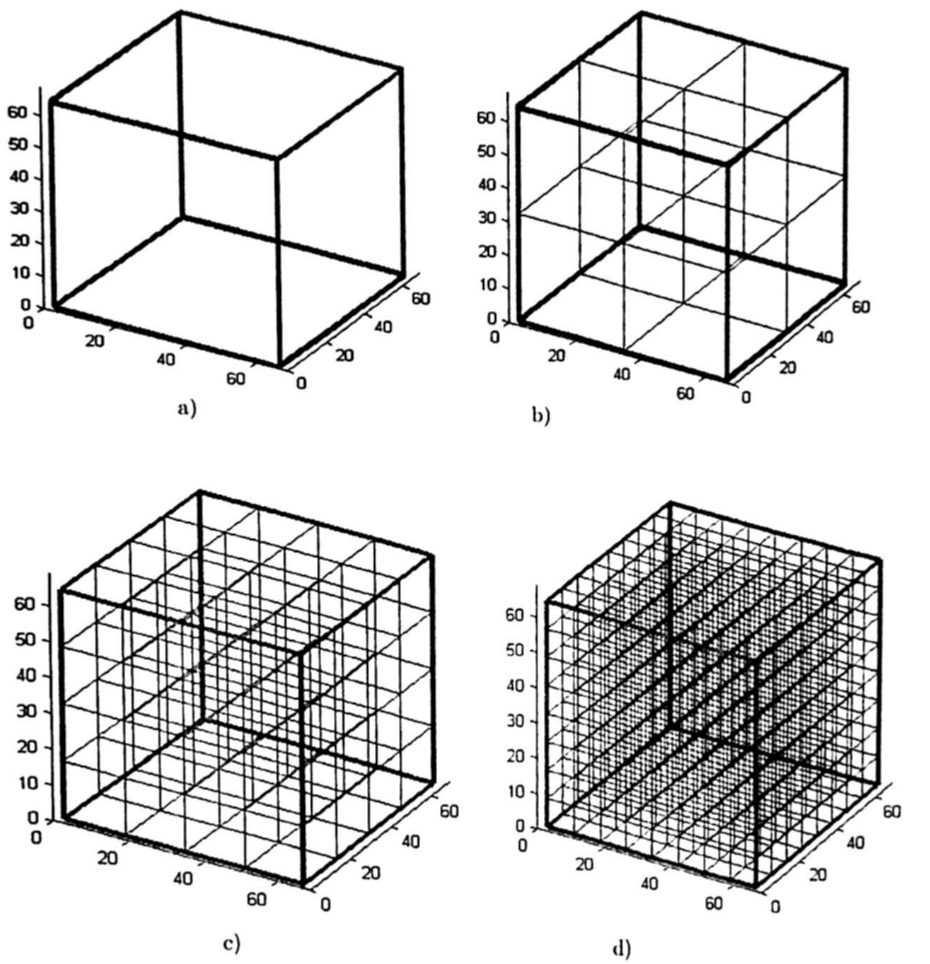

Fig. 11 a) $r_0 = 1$, $N\left(\frac{1}{r_0}\right)^3 = 1$, b) $r_1 = \left(\frac{1}{2}\right)$, $N\left(\frac{1}{r_1}\right)^3 = 8$, c) $r_2 = \left(\frac{1}{4}\right)$, $N\left(\frac{1}{r_2}\right)^3 = 64$, d) $r_3 = \left(\frac{1}{8}\right)$, $N\left(\frac{1}{r_3}\right)^3 = 512$

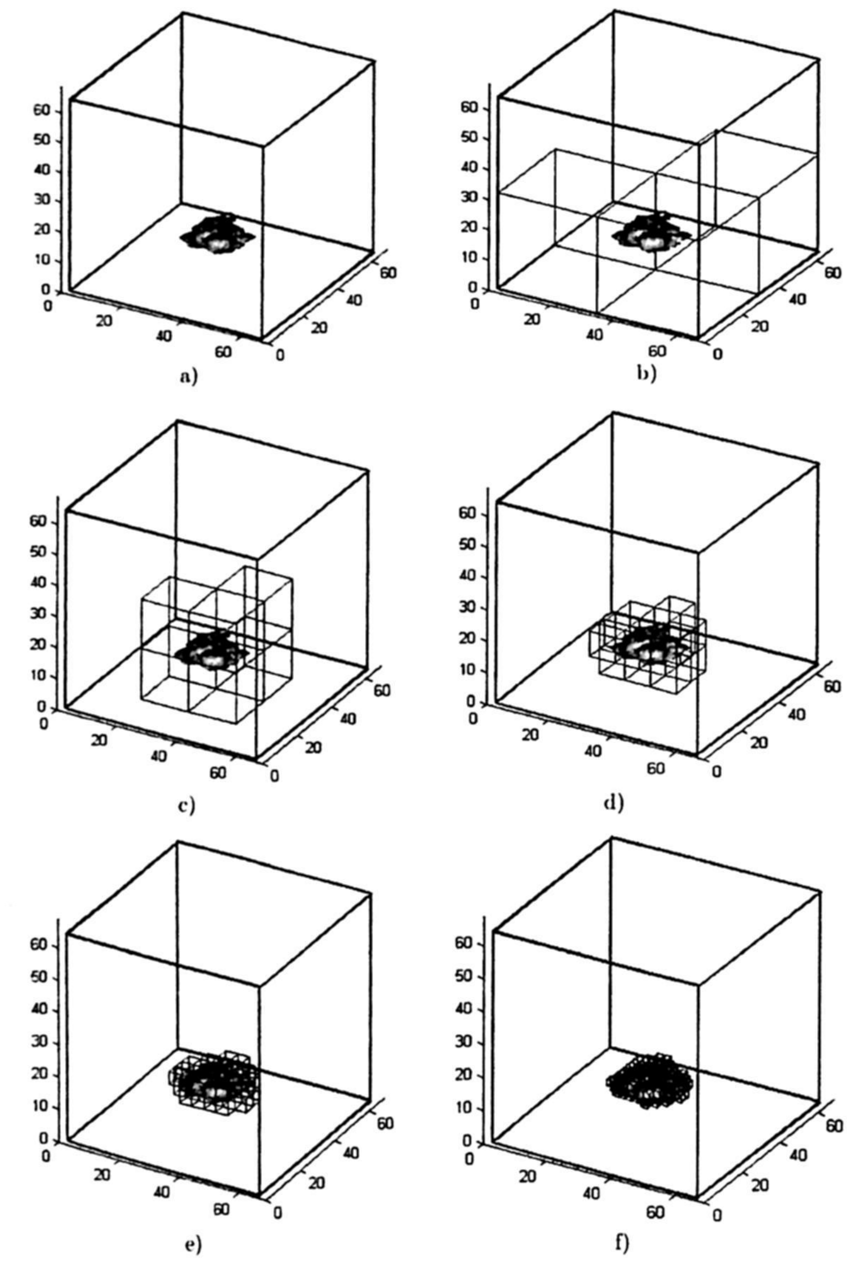

Fig. 12 Stages of the measurement of the tumor

In this research, six iterations were used to measure the brain tumor. Table 2 shows the results of the six test iterations. The results confirm that more precision is gained with smaller values of r.

Table 2. Iterations for measuring the tumor

Iteration	n	r	N(r)	Boxes that contain the tumor
1	64	1	1/(1/1) ^ 3 = 1	1
2	32	1/ 2 = 0.5	1/(1/2) ^ 3 = 8	3
3	16	1 / 4 = 0.25	1/(1/4) ^ 3 = 64	6
4	8	1 / 8 = 0.125	1/(1/8) ^ 3 = 512	15
5	4	1 / 16 = 0.0625	1/(1/16) ^ 3 = 4096	42
6	2	1/ 32 = 0.03125	1/(1/32) ^ 3 = 32768	183

The fractal dimension is obtained by using equation 7 and the results are presented in Table 3.

Table 3 Fractal dimension of the tumor.

m = *fractal dimension*	2.4378
B	5.9124

5 Conclusions

The measurement of a brain tumor with Box-Counting in a 3-D space is proposed in this paper. The Box-Counting method requires a prior segmentation process [1, 2, 3] and reconstruction process [7, 8]. The Box-Counting method does not depend on scale of the image as some other methods do. Box-Counting's precision depends on the iteration number. A large iteration number will imply smaller r value, smaller box size, better precision and greater computer resources required.

The fractal dimension obtained (d=2.4378 in test example) provides important information about the position and size of the tumor. This information helps the oncologist to make objective decisions about diagnostics and treatment of the condition.

References

1. Cortés P. Ernesto, "Detección de Cáncer Cerebral: Mediante Computación Inteligente", Memorias del Congreso Internacional de Informática y Computación, ANIEI 2006, XIX, Tuxtla Gutiérrez, Chiapas, Octubre 23-25, ISBN 970-31-0751-6.
2. Cortés P. Ernesto, "Metodología para la extracción de características en imágenes medicas: Tomografías Computarizadas y Resonancias Magnéticas Cerebrales", Memorias del Congreso MICAI 2006 Fifth Mexican Internacional Conference on

Artificial Intelligence, en el Workshop de Tecnologías Inteligentes, Apizaco Tlaxcala, México, Noviembre 13-17, 2006, ISBN 970-94214-1-7.

3. El-Gohary, Awad "Chaos and optimal control of equilibrium states of tumor system with drug" Chaos, Solitons & Fractals (2008), doi:10.1016/j.chaos.2008.02.003
4. El-Gohary, Awad "Chaos and optimal control of cancer self-remission and tumor system steady states" Chaos, Solitons & Fractals, 2008;37(5):1305-16
5. Falconer, K. J. "Techniques in Fractal Geometry" John Wiley & Sons Ltd, 1997
6. Falconer, K. J. "Fractal Geometry-Mathematical Foundations and applications" John Wiley & Sons Ltd, Chichester 1990
7. Canny, John, "A Computational Approach to Edge Detection," IEEE Transactions on Pattern Analysis and Machine Intelligence,Vol. PAMI-8, No. 6, 1986, pp. 679-698.
8. Barber, C. B., D.P. Dobkin, and H.T. Huhdanpaa, "The Quickhull Algorithm for Convex Hulls," ACM Transactions on Mathematical Software, Vol. 22, No. 4, Dec. 1996, p. 469-483.
9. National Science and Technology Research Center for Computation and Visualization of Geometric Structures (The Geometry Center), University of Minnesota. 1993.
10. Lim, Jae S., Two-Dimensional Signal and Image Processing, Englewood Cliffs, NJ, Prentice Hall, 1990, pp. 478-488.
11. Parker, James R., Algorithms for Image Processing and Computer Vision, New York, John Wiley & Sons, Inc., 1997, pp. 23-29.
12. Otsu, N., "A Threshold Selection Method from Gray-Level Histograms," IEEE Transactions on Systems, Man, and Cybernetics, Vol. 9, No. 1, 1979, pp. 62-66.
13. Construcción de mallas Delaunay en tres dimensiones, Enzo A. Dari, Centro Atómico Bariloche, Comisión Nacional de Energía Atómica, Instituto Balseiro, Universidad Nacional de Cuyo y C.N.E.A Consejo Nacional de Investigaciones Científicas y Técnicas.
14. "Triangulacion Delaunay", *Danie Martos López, Ricardo Navarro Moral,* Universidad de Jaén, Departamento de Informática y Geometría Computacional.
15. Zadeh, L. Fuzzy logic, IEEE Computer, 1:83, 1988.
16. Clasificación de Llanto del Bebé Utilizando una Red Neural de Gradiente Conjugado Escalado, J. Orozco García, Carlos A. Reyes García, Instituto Nacional de Astrofísica Óptica y Electrónica.
17. VISIÓN POR COMPUTADOR, Imágenes digitales y aplicaciones, Gonzalo Pajares, Jesús M. de la Cruz, ed. RA-MA.
18. González, R., Woods, R. Digital Image Processing. Adison – Wesley.
19. Li, W. "An equivalent definition of packing dimension and its application" Nonlinear Analysis: Real World Applications. Doi:10.1016/j.nonrwa.2008.02.004